# Under-Water Molecular Signalling: a Hidden Transmitter and Absent Receivers Problem

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*Abstract*—**Wave-based signals have been successful in reliably and efficiently transferring data between two or more well defined points (e.g., known location area). However, it is challenged when the transmitter is hidden and the receivers are absent. Essentially, the transmitter and the receivers have no location knowledge of each other. We demonstrate that unlike wave-based transmissions, the total molecular energy doesn't monotonically degrade as a function of time.**

**This paper uses a bio-inspired method of communicating data from a hidden transmitter to a group of absent receivers. A specialized molecular communication system is designed, including how to embed vital location information in the structure of a heterogeneous biochemical molecule. Like *message in a bottle*, there is a growing probability of receiving the location message over a period of several years. The only caveat is that there is an initial delay of a few hours to days, depending on the proximity of the rescue team to the crash site. This will provide an attractive alternative to current wave-based communications for delay-tolerant crash recovery.**

## I. INTRODUCTION

For the better half of the last century, wireless communications has been dominated by waveform-based information carriers, transferring energy from a transmitter to one or more receivers. On land, electromagnetic (EM) wave-based communications is ubiquitous to modern life, with over 2 million base stations serving some 8 billion mobile handsets. This success is only widespread in well-known environments, where radio planning is possible. Challenges remain when we want to transfer information in environments where the transmitter and receivers have no knowledge of each others location areas, and the propagation of EM- and acoustic-waves is severely attenuated by the channel (i.e., under-water).

### A. HTAR Problem

In this particular paper, we address the challenge of locating a hidden object in the ocean (i.e., a submarine disaster or plane crash). The location area of the object is unknown and the search radius can be up to 1000km, as is the case for the AF447 (2009) and the MH370 (2014) air crashes. This search radius far exceeds the typical communication range underwater

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($\sim 10 - 30$km) for any current technology. This localization problem has two distinctive characteristics:
1) a hidden transmitter,
2) absent receivers from the vicinity of the transmitter.

We call this the hidden transmitter and absent receiver (HTAR) problem. Knowledge of either where the transmitter is, or presence of the receiver in the vicinity of the transmitter within a set time frame would solve the localization problem. The time frame constraint arises from the finite energy of transmitters.

### B. Contribution and Organisation

Our contribution is to propose a molecular based signalling system [1], where information about the transmitter's location is encoded within the chemical composition of the molecules. To an extent, this is akin to nature, where moths are attracted to each other through pheromone signalling that diffuses in the wind [2]. We demonstrate that unlike wave-based transmissions, the total molecular energy doesn't monotonically degrade as a function of time. Like a *message in a bottle*, there remains a growing probability of receiving the location message over a period of several years. The only caveat is that there is an initial delay of a few hours to days, depending on the proximity of the rescue team to the crash site. This will provide an attractive alterative to current wave-based communications for *delay-tolerant crash recovery*.

## II. REVIEW OF CURRENT SYSTEMS

### A. Acoustic Communications (AC)

Current *black box* and other underwater communications utilize acoustic wave systems known as Underwater Locator Beacon (ULB) to transmit information in the form of 10ms sharp pulses (pings) on a 37.5 kHz carrier frequency. A typical battery supply can last up to 30 days, and current receiver technologies (180dB and $1\mu$Pa) can reliably detect the signal at a range of 5km (normal conditions) and 7km (good conditions) [3]. The fundamental problem with all wave-based communications is that once the signal pulse is transmitted, the pulse's energy decays with propagation distance over time. For example, in an underwater acoustic channel over a propagation distance $R$ [km], the propagation channel's energy attenuation $A$ [dB] is statistically characterised by [4]:

$$A_{\text{AC}}(R, f) = k10 \log_{10}(R) + R(a(f)) + 10 \log_{10}(A_0), \quad (1)$$

where $A_0$ is a constant, and $k$ is an acoustic spreading factor (typically 2). The function $a(f)$ characterises the absorption
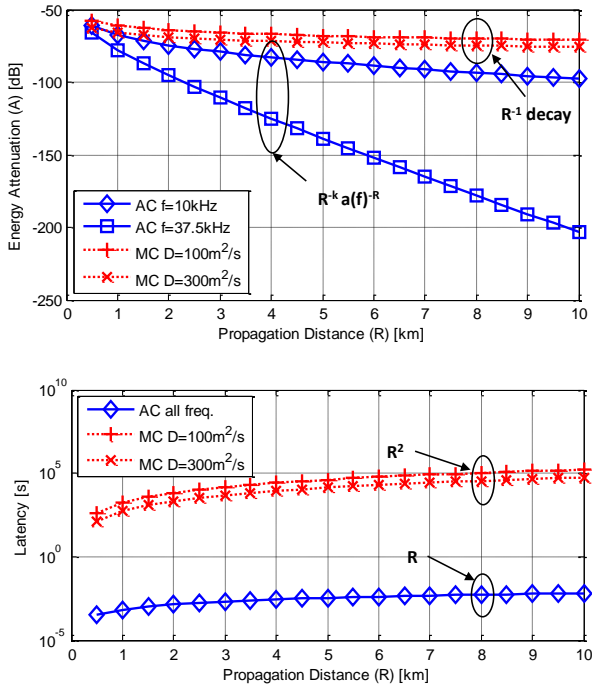
Fig. 1. Plot of acoustic and molecular energy attenuation (top) and latency (bottom) as a function of distance $R$.

coefficient which is a function of frequency $f$ [kHz]. The time delay to peak amplitude's arrival is $\propto R$.

### B. Molecular Communications (MC)

On the other hand, molecular communication systems utilize information carrying molecules to convey messages. A prototype that can communicate text messages in real time over macro-scale distances [5]. Our proposed system is inspired by this innovation, but differs in that it repeatedly sends the same message (location), and modulates in the chemical composition domain [6], as opposed to the amplitude domain.

Molecular diffusion in 3-dimensional space, can be characterized by the hitting probability density function:

$$\phi_{\text{MC}}(R, D, t) = \frac{1}{(4\pi Dt)^{3/2}} \exp\left(-\frac{R^2}{4Dt}\right), \qquad (2)$$

where $D$ is the diffusivity, which can be an empirical value that takes into account of flow and hence take on the dimension of being non-isotropic and time variant ($D(x, y, z, t)$).

As pointed out in [7], the received molecular energy can be considered as the total number of molecules accumulated over time. For simplicity, we initially consider a single narrow pulse transmission and an infinite time reception. Using this definition, the MC channel's energy attenuation can be expressed as a function of propagation distance $R$:

$$A_{\text{MC}}(R, D, t) = \frac{1}{4\pi DR}, \qquad (3)$$

where the time delay to peak arrival is $\propto R^2$.

### C. Impulse Response Comparison

Initially, we consider the energy of the impulse response for an acoustic-wave communication (AC) and a molecular communication (MC) system. The results in Fig. 1(top) show that molecular energy decays at $R^{-1}$, as opposed to $R^{-2}a(f)^{-R}$ for acoustic waves. This means rescue receivers are far more likely to detect molecular messages at long distances than acoustic waves, even at very low frequencies of 10kHz. The results in Fig. 1(bottom) show that the peak molecular pulse's arrival time (latency) increases quadratically with distance ($R^2$), whereas acoustic waves' peak time increases linearly ($R$) [7]. This means rescue receivers need to wait significantly longer. Typically, the waiting time for an acoustic wave at 10km is 10ms, whereas for a molecular pulse (with ocean currents) is several hours.

Given that acoustic receiver has a certain sensitivity threshold, there is both a finite distance and time (energy supply), beyond which the receiver cannot reliably receive the signal. Therefore, the absence of the receivers from the reception zone in that time period will mean the transmitters location is lost. In order to solve the HTAR problem, the transmitter must send messages that can persist for a long period and over long distances. Hence, we are motivated to propose an alternative system based on chemical messaging.

## III. SYSTEM DESIGN

### A. Chemical Information Carriers

*1) Small Molecules:* Small Chemical molecules such as hormones or pheromones can be sent at short or long ranges. Unlike all macro-biochemicals, which will be quickly digested in the environment, small molecules are relatively stable in most fluid- and gas-phase channels and can yield up to $10^6$ unique chemical combinations. In terms of real time generation, there are two routes. Biologically generated messages can be achieved by using a biological entity to transform a physical input (e.g., electrical current, or light) into a chemical compound response. The challenge with small molecule messages is that each message has 1 biological receptor and therefore to differentiate the messages will require very complex transmitter and receivers. As the message is a fairly simple chemical, it is possible to use a chemical receiver rather than a biological one [8], which is the second route. However, the complexity of the receivers generally restrict detection to a selective small set of chemical compositions.

*2) Peptides or Amino Acids:* Peptide molecules can transport a higher information content as each amino acid has 20 variables (usually, more in various microbes), yielding a near limitless complexity. There are natural receptors in the form of antibodies one can use to translate the message quickly and possibly bypassing a biological receiver. However, any peptide of >10 amino acids will likely have secondary and tertiary structures so some parts of the message may get hidden on the inside. Creating a peptide sequence ad hoc is very challenging as chemical synthesis is slow and struggles with any peptide over 50 amino acids, and doing so biologically is basically
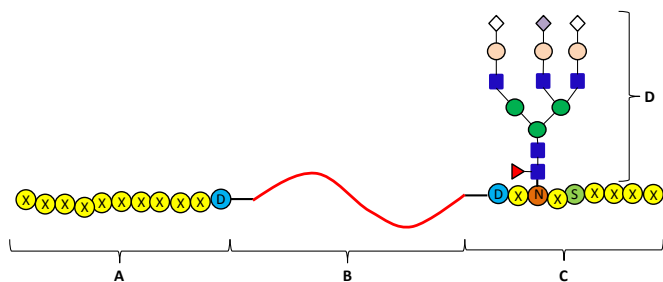
Fig. 2. An example of a possible chemical message, comprised of peptides, non-biological molecules, and a N-linked glycan.

impossible as all peptides are produced based on a pre-existing RNA sequence.

*3) DNA:* DNA is rarely, if ever sent in nature as a form of information carrier. However, there are small circular DNA molecules called plasmid which transfer themselves between microbes [9]. Unfortunately, the mechanism by which they do this is very poorly understood, but does work at very short distances. Modern DNA sequencing technology can decode gigabase sequences in a matter of hours [1], but synthesising de novo DNA messages of any significant length is slow and difficult chemically, and biologically impossible.

*4) Carbohydrates:* Carbohydrates are the class of biological macromolecule with probably the greatest potential for complexity and therefore for storing information. Monosaccharides typically contain 3 to 6 carbons, and many can exist as both ringed and straight-chained molecules. Monosaccharides can vary in relatively subtle ways [10], which makes them very difficult to tell apart chemically as well as by mass, but modern technology have made it possible to map their sites and structures [11]. Unlike DNA and peptides, they can polymerise in more than one dimension as glycosidic linkages can be formed between multiple sites of monosaccharides, making a specific polysaccharide structure extremely difficult to synthesise chemically robustly. However, unlike DNA or peptides, they do not require a template to be produced biologically. Recent advances has made in vitro production of specific polysaccharide structures a reality [12], therefore polysaccharides or glycoprotein messages can be both produced and interpreted.

### B. Message Structure

It may be useful to use a combination of biological and non-biological molecules to assemble a chemical message. Although biological molecules hold greater potential for complexity, it is usually faster and more reliable to synthesise non-biological polymers. Modern biochemical engineering can use a range of techniques for modifying biomolecules with a range of chemical groups (such as with dehydroalanine [13]) which can be bonded with any number of non-biological molecules. Therefore each system can have its own unique

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[1]An example system can be found at: http://systems.illumina.com/

peptide sequence, ending in a functionalised dehydroalanine, which can be assembled with 3 different polymers in sequence each giving a different dimension of the coordinates. A second peptide containing a glycosylation site could then be added with a customised carbohydrate using different glucosyl transferases and hydrolases to create a personalised message. In Fig. 2, we illustrate an example of a possible chemical message, with the following components:

- A: peptide epitope signature specific to the system that can be easily recognised by a specific antibody. X = any amino acid, D = dehydroalanine, used to connect the peptide to non-biological molecules.
- B: non-biological components of different chemical makeups of specific mass to give accurate location data.
- C: peptide with N-glycosylation recognition motif Asn-X-Ser to allow glycosylation.
- D: A typical mammalian N-linked glycan with each different colour and shaped object representing a different type of monosaccharide. Glycans can vary in size, structure and make up and can be customized using different monosaccharides and enzymes during synthesis, making it a good candidate molecule for personalized messages.

The whole message would then be wrapped in a non-biodegradable polymer by a bubble machine, and propelled into the environment.

### C. Transmitter and Receiver Design

An envisaged chemical message transmitter would have 3 sections. Section 1 would be connected to the navigational equipment of the vehicle. Section 2 would be the message synthesis system, which would use the information from section 1 to create the chemical message. This section would need to have a numerous of compartments to house the various different possible components of a message, the catalysts for assembling the message, and reaction chambers for the messages to be synthesised. Numerous reaction chambers are probably necessary for speedy synthesis of different components of a message before final assembly. Section 3 would be the message delivery system which would package the message in a shell and propel it out of the vehicle. We envisage a micro-bubble machine that can produce bubbles that act to both protect the chemical information carriers and can be detected by a receiver. A receiver would first have to identify and unwrap a message; it would then reverse the assembling mechanism to dissect the message into its component parts. The peptide identification components may be quickly identified by antibody based techniques. Glycosylated peptide and polymers could both be interpreted using LC-MS [11].

## IV. DIFFUSION PROCESS IN OCEAN

### A. 3D Diffusivity

This paper considers molecular communications in underwater ocean environment in the context of the HTAR problem. Consider a flight crash into the ocean (71% of the earth's surface) and sinks to the bottom. We set the location of the flight as the origin of the crash coordinates. The underwater
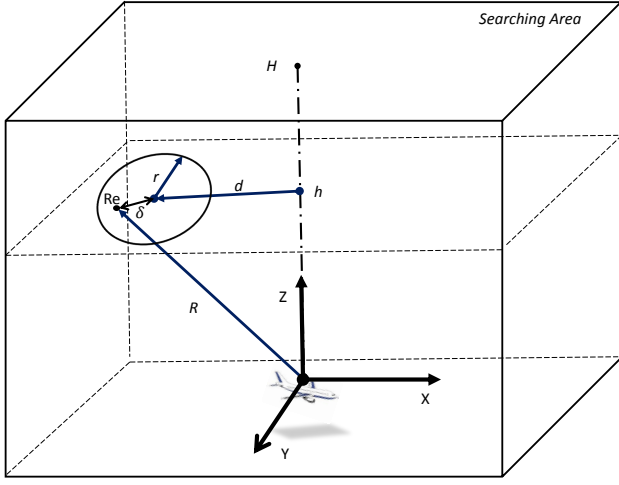
Fig. 3. Illustration of HTAR problem for flight crash rescue modelling.



Fig. 4. Plot of MC's pulse response at different $d$ values.

diffusion propagation model has an origin at the bottom of the ocean where the plane is, as shown in Fig. 3. We assume that the molecules are of the same density as water and the vertical forces exerted to the molecules are entirely related to diffusion and ocean currents.

The propagation process of molecules released at the origin can be modeled by solving Fick's laws of diffusion. Since the molecules can only propagate in a hemisphere, therefore, if the molecules released at the time instant $t = 0$, the impulse response (hitting probability density function) $\phi$ at a given point $(x, y, z)$ in the propagation model is given as:

$$\phi(x, y, z, t) = \frac{2 \exp\left(-\frac{x^2}{4D_x t} - \frac{y^2}{4D_y t} - \frac{z^2}{4D_z t}\right)}{(4\pi t)^{3/2} \sqrt{D_x D_y D_z}}, \quad (4)$$

where $D_x$, $D_y$ and $D_z$ are the diffusivity of $x, y, z$ directions respectively. Unlike most environments, the diffusivity is not isotropic in oceans and therefore we consider specific ocean diffusivity values. In oceans, empirical results have shown that diffusivity of both vertical ($z$) and horizontal ($x, y$) directions depend on the ocean depth [14].

### B. Pulse Response and Energy

Previously in Section II, we considered an impulse response with infinite time reception. We now consider a finite pulse of duration $T$ and a finite receiver time frame of $T_2 - T_1$. Let us consider a transmitter that emits molecules continuously from $t = 0$ to $t = T$, which can be modelled as a rectangular pulse with amplitude $M$.

For a step input with a delay $\tau$ and magnitude $M$ (molecules per second), the step response $S(x, y, z, t, \tau)$ at a given point $(x, y, z)$ is a convolution of the delayed step input and the impulse response $\phi(x, y, z, t)$. We present $S(x, y, z, t, \tau)$ in Eq. 5 (derivation shown in the Appendix):

$$S(R', t, \tau) = \frac{M}{2\pi D R'} \operatorname{erfc}\left(\frac{R'}{2\sqrt{D(t - \tau)}}\right), \quad (5)$$
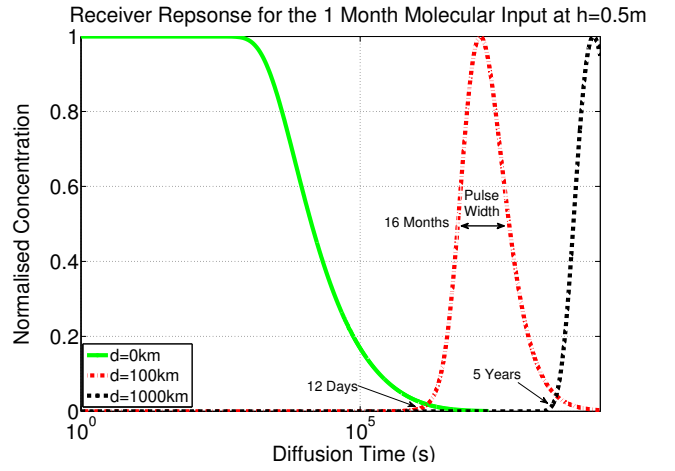
where $D$ is the equivalent diffusion coefficient given as $D = (D_x D_y D_z)^{\frac{1}{3}}$, $R'$ is the equivalent molecular propagation distance given as $R' = \frac{1}{D}(D_x D_y z^2 + D_x D_z y^2 + D_y D_z x^2)^{1/2}$ and erfc () is the complementary error function.

For a finite pulse duration of $T$, the pulse response $P$ can be expressed as $S(x, y, z, t, \tau = 0) - S(x, y, z, t, \tau = T)$. We plot the pulse response $P$ in Fig. 4 as a function of different distances $d$ from crash location at water level.

Previously, the molecular energy has been defined as the total number of molecules received over an infinite time [7]. In reality, the receiver can not sample from $t = 0$ to $t = +\infty$. We define a finite period energy, which is $E_p = \int_{T_1}^{T_2} P(R', t, \tau)\,dt = E_s(R', t, \tau = 0) - E_s(R', t, \tau = T)$, where $E_s$ is given in the Appendix.

### C. Receiver Distribution

We consider receivers distributed in a circular field of radius $r$ at a distance $d$ from the ocean surface of the crash site. Two kinds of receiver deployment spatial distributions are considered, namely:

1) Normal Random Deployment (NRD)
2) Uniform Random Deployment (URD)

Within the circular receiver region, we define the random distance between the centre to a receiver $i$ as $\delta_i$, which follows a distribution $f_D(\delta)$:

$$f_D(\delta) = \begin{cases} \frac{1}{2r} & \text{URD} \\ \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\delta^2}{2\sigma^2}} & \text{NRD} \end{cases}, \quad (6)$$

where $\sigma$ is the standard deviation.

We deploy $N$ molecular receivers in the deployment area and specify the location of each receiver $(x_i, y_i, z_i)$ which follows the distribution of the aforementioned deployment methods. Therefore, the total molecular received energy $E_{p,\text{total}}$ from $N$ molecular receivers can be expressed as the sum of

TABLE I
THE DEFINITION OF PARAMETERS IN THE MODEL

| Parameters | Symbols [Units] |
|---|---|
| **Acoustic System** | |
| Frequency | $f$ [kHz]: 10, 37.5 |
| Tx Power | $P_{ac}$ [W]: 0.65 |
| **Molecular System** | |
| No. Emitted Molecules per s | $M$: 1 Mol. ($6 \times 10^{23}$) |
| Transmission Period | $T$ [s]: $0 - 2 \times 10^6$ |
| Diffusion Coefficient | $D$ [m$^2$/s] |
| Total Pulse Energy | $E$ [mol s$^2$/m$^3$ ] |
| Received Pulse Energy | $E_{p,\text{total}}$ [mol s$^2$/m$^3$ ] |
| **Propagation Environment** | |
| Search Area | $7.68 \times 10^6$ km$^2$ |
| Average Sea Depth | $H$ [km]: 5 |
| Search Level | $h$ [m]: $0 - 500$ |
| Rx Deployment Radius | $r$ [km]: $0.5 - 50$ |
| No. of Receivers | $N$: $10 - 1000$ |
| Propagation Distance | $R$ [km] |
| Receiver Start Time | $T_1$ [s]: $8 \times 10^4$ |
| Receiver Finish Time | $T_2$ [s]: $2 \times 10^8$ |

individual receivers' energy $E_{p,i}$:

$$E_{p,\text{total}} = \sum_{i=1}^{N} E_{p,i}(R', t) = \frac{N}{2\pi r} \int_0^r \int_0^{2\pi} E_{p,i} f_D(\delta)\, \mathrm{d}\theta \mathrm{d}\delta, \quad (7)$$

where $\theta$ is the angle that the receiver makes to $d$.

## V. RESULTS AND ANALYSIS

In this section, this paper will now consider a number of scenarios for underwater rescue, and compare the performance of AC and MC systems. The parameters used to plot the following results can be found in Table.I.

*1) Long Endurance:* In Fig.5, we demonstrate the received energy over a finite time, for the conventional AC and the proposed MC system, both transmitting for the same duration of 1 month and at the same propagation distance of $R = 20$km. From the results, it can be seen that the received energy of the AC system is restricted significantly by the transmission period, whereas the chemical messages will persist in the ocean for significantly longer, rising in energy over time (at least up to a few years).

*2) Deployment Formation:* In Fig.6, we demonstrate the effect of the two deployment methods on MC's received energy. The results show that for a medium or longer distance away from the crash site ($d > 30$km), the deployment formation is not important. For smaller distances of a few km, the NRD formation with a small standard deviation of $\sigma < 11$km will perform significantly better than URD formation. However, a high standard deviation of $\sigma > 11$km will perform worse. For example, NRD with a standard deviation of $\sigma < 180$m will yield a 35% increase in energy received; whilst NRD with a standard deviation of $\sigma = 18$km will yield a 17% decrease in energy received. This is intuitive, as the closer the receivers
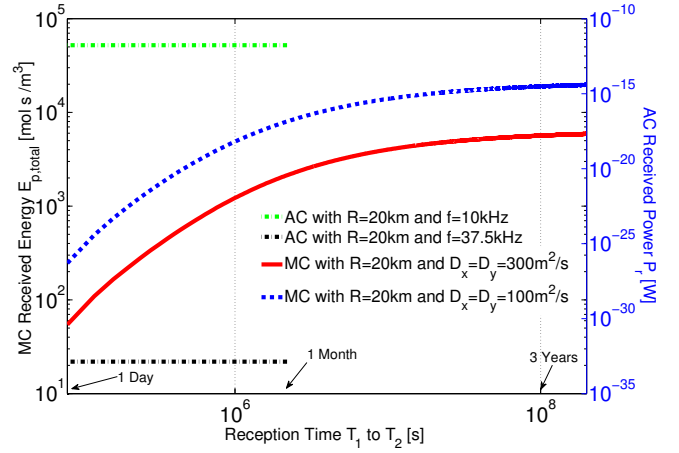


Fig. 5. The comparison between AC and MC systems at a set receiver location over a finite reception time of 1 day to a few years. Parameters: $r = 18$km, $h = 0.5$m, $d = 20$km, $R = 20$km, $D_z = 5 \times 10^{-5}$m$^2$/s, $\sigma = r/100$, $T_1 = 40000$s (12 hours), and $T_2 = 80000$s (1 day) to $2 \times 10^8$s (6 years).
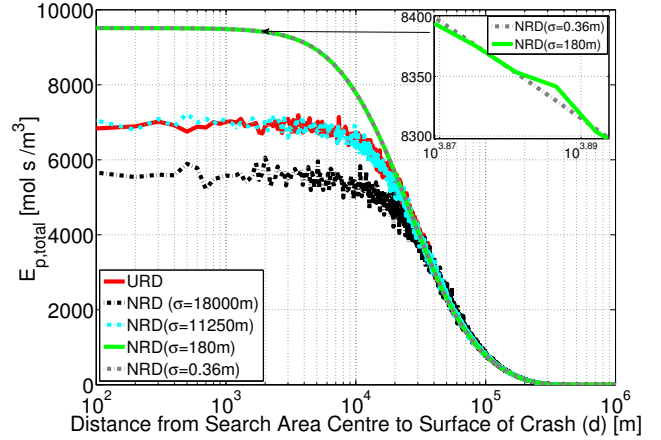


Fig. 6. The comparison between NRD with different $\sigma$ and URD over an increased $d$. Parameters: $r = 18$km, $N = 100$, $h = 0.5$m, $D_x = D_y = 300$m$^2$/s [14], $D_z = 5 \times 10^{-5}$m$^2$/s, $T_1 = 40000$s (12 hours) and $T_2 = 10^8$s (3 years).

are to the crash site, the more focused their formation needs to be towards the centre of the search area.

*3) Receiver Deployment Area:* In Fig. 7(a), we demonstrate the effect of receivers' deployment radius $r$ on energy. The results show the received energy is independent of the deployment distribution when $r < 10$km. The parameters used are: $N = 100$, $h = 0.5$m, $d = 100$km, $D_x = D_y = 300$m$^2$/s [14], $D_z = 5 \times 10^{-5}$m$^2$/s, $T_1 = 40000$s, $T_2 = 10^8$s. The results show that for larger deployment radius, NRD with a large variance is preferred, which is intuitive given that this increases the probability that a given receiver is closer the unknown crash site.

*4) Search Depth:* In Fig. 7(b), we demonstrate the effect of the searching depth distance $h$ on the received energy in comparison with $d$. The parameters used are: $r = 18$km, $N = 100$, $D_x = D_y = 300$m$^2$/s, $D_z = 5 \times 10^{-5}$m$^2$/s,
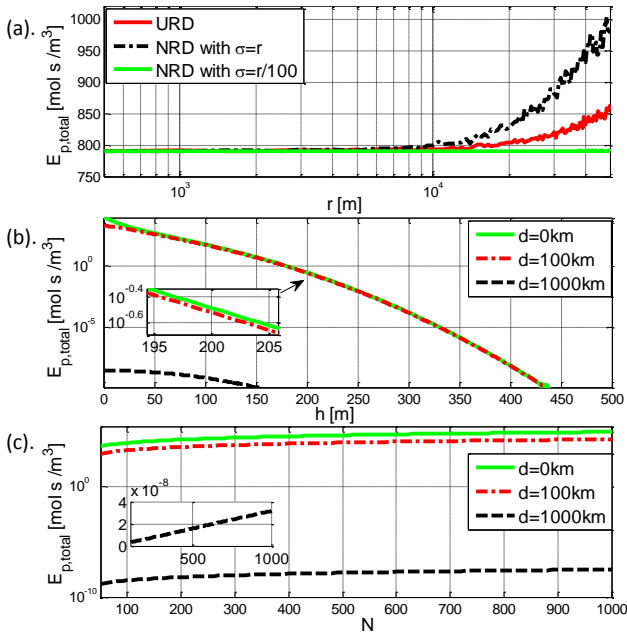
Fig. 7. Plots of the relationship between energy $E_{p,\text{total}}$ and the parameters $r, h, N$ respectively.

$T_1 = 40000\text{s}$, $T_2 = 10^8\text{s}$ and deployment formation: NRD with $\sigma = r/100$. The results show that the receiving energy is very sensitive to $h$, whereby increasing every 100m can reduce $E_{p,\text{total}}$ by several orders of magnitude. Therefore, a rationale conclusion is that $h$ must be small which means the molecular receivers must be deployed as deep as possible.

*5) Effect of Number of Deployed Receivers:* In Fig. 7(c), we demonstrate the effect of the number $N$ of receivers on the received energy. The parameters used are: $h = 0.5\text{m}$, $r = 18\text{km}$, $D_x = D_y = 300\text{m}^2/\text{s}$, $D_z = 5 \times 10^{-5}\text{m}^2/\text{s}$, $T_1 = 40000\text{s}$, $T_2 = 10^8\text{s}$ and deployment formation: NRD with $\sigma = r/100$. It is clear that received energy has a linear growth with $N$ from the results and from Eq. 7.

## VI. CONCLUSION

In this paper, we have proposed molecular communication which has advantages on persistent and long endurance for the Hidden Transmitter and Absent Receiver (HTAR) problem. In particular, we consider finding a crashed object at the bottom of an ocean. A specialized molecular communication system is designed, including how to embed vital location information in the structure of a heterogeneous biochemical molecule. We concluded that acoustic communications is not viable due to the short range of acoustic waves ($< 20\text{km}$), and the finite battery life of most transmitters (1 month). On the other hand, molecular communications with chemical modulation is a viable solution to the HTAR problem, able to diffuse long distances ($\sim 1000\text{km}$) and achieve long endurance ($\sim$ years). The caveat is that an initial few hours to days is needed before the first molecules arrive at receivers, which is reasonable for delay tolerant crash recovery.

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## APPENDIX

We consider the Laplace transform of the channel impulse response $\phi_{\text{MC}}(R, D, t)$ is:

$$\mathcal{L}_t \left[ \frac{2}{(4\pi Dt)^{3/2}} \exp\left( -\frac{R'^2}{4Dt} \right) \right] = \frac{e^{-R'\sqrt{\frac{s}{D}}}}{2\pi DR'}. \quad (8)$$

Therefore, a step response with delay $\tau$ is an inverse Laplace transform of $\exp(-\tau s)/s \times$ Eq. 8:

$$S\left( R', t, \tau \right) = \frac{M}{2\pi DR'} \text{erfc}\left( \frac{R'}{2\sqrt{D(t - \tau)}} \right). \quad (9)$$

The energy of the step response over a finite time period is:

$$E_s(R', t, \tau) = \int_{T_1}^{T_2} S\left( R', t, \tau \right) \text{d}t = \frac{M}{2\pi DR'} \times$$
$$\left[ t'\{\text{erfc}(\frac{R'}{2\sqrt{Dt'}}) - \frac{R'e^{-\frac{R'^2}{4Dt'}}}{\sqrt{\pi Dt'}}\} - \frac{R'^2\text{erf}(\frac{R'}{2\sqrt{Dt'}})}{2D} \right]\Big|_{T_1}^{T_2}, \quad (10)$$

for $t' = t - \tau$.